Activity 1 – Characteristics of Binary Stars
(from http://spiff.rit.edu/classes/phys301/lectures/mass_ii/mass_ii.html &

Objectives
• To determine the mass of a binary star system.
• To investigate the effect that changes in mass, separation, and eccentricity, have on the radial velocity (and hence orbital period) of a spectroscopic binary.

Introduction
The most important property of a star is its mass, but stellar masses are much harder to measure than luminosities or surface temperatures. The most dependable method for "weighing" a star relies on Newton’s version of Kepler’s third law. This law allows us to calculate the masses of orbiting objects by measuring both the period and average distance (semimajor axis – see diagram) of their orbit.

There are three main classes of binary systems: visual binary, spectroscopic binary, and eclipsing binary. In the first exercise, we examine the characteristics of Procyon, a spectroscopic binary system.

If we look at Procyon with a telescope (see image), we see a very bright star of magnitude 0.4; and, if we look very carefully, we can make out a much fainter star nearby, roughly magnitude 11. This picture was taken by a NASA telescope in 1995. The “window frame” is a screen that blocks light from the brighter star so that the dimmer one (in the square) will show up in the exposure.

This very faint companion (Procyon B) was first observed by Schaeberle at Lick Observatory in 1896. At that time, the position of Procyon B has been recorded to follow its motion around the bright star (Procyon A).
Exercise 1.1
Q: What is the period of Procyon's orbit?

\[ P = 41 \text{ years or so} \]

Q: What is the apparent angular size of the semi-major axis?

About \( \theta = 4.1 \text{ arcseconds} \)

Exercise 1.2
To determine the mass of the system, we will apply Kepler’s Third Law. The required variables are: period of the system and the actual linear size of the semi-major axis (the approximate radius) of the orbit. Given the parallax of Procyon to be 0.286":

Q: What is the distance to Procyon?

\[
\frac{1}{d} = \frac{1}{0.286 \text{ arcsec}} = \frac{1}{0.286} = 3.50 \text{ pc}
\]

Q: What is the linear size of the semi-major axis?
Express your answer in Astronomical Units (AU).

\[
a = (\theta \text{ in arcsec}) \times (d \text{ in pc})
\]

\[
= (4.1 \text{ arcsec}) \times (3.50 \text{ pc})
\]

\[= 14 \text{ AU}\]

(You can also convert all units to meters and degrees, compute the linear size of the semi-major axis in meters, then convert back to AU at the end)
Exercise 1.3
Kepler’s third law (not in the syllabus) can be used to determine the mass of two bodies in an orbiting system.

\[ \frac{T^2}{r^3} = \frac{1}{M} \]

- \( T \) = period (in years)
- \( r \) = semi major axis of orbit (in AU)
- \( M \) = mass of system (in \( M_\odot \))

Use this equation to determine the mass of the Procyon binary system.

Q: What is the total mass of both stars in the Procyon system, in terms of solar masses?

\[ \frac{3}{(\text{semi-major axis in AU})^3} \times \frac{1}{(\text{Period in years})^2} \]

In round numbers,

\[ \frac{(14 \text{ AU})^3}{(41 \text{ yr})^2} \approx 1.6 \text{ solar masses} \]

Note that this calculation assumes that the orbital plane of the star system (diagrammed previously) is perpendicular to our point of view.

In reality, the orbital plane is inclined. The inclination angle is measured from the orbital path to a line perpendicular to the position of the observer (this would be the orbital plane in the diagram).
How do we know that the orbit is inclined? We can examine this using the *Orbiting Binary Stars* computer simulator (link can be found on the course WIKI).

<table>
<thead>
<tr>
<th>M1 or M2</th>
<th>The mass of each of the two stars.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Separation a:</td>
<td>The distance between the two stars in solar radii.</td>
</tr>
<tr>
<td>Eccentricity e:</td>
<td>Eccentricity of the orbit</td>
</tr>
<tr>
<td>Inclination angle i:</td>
<td>Angle of the orbital plane of the stars to our line-of-sight. 0° - face on</td>
</tr>
<tr>
<td>Node angle w: (ignore)</td>
<td>Angle of the major axis as measured in the orbital plane. This affects the privileged view and we will not change it in the course of our study.</td>
</tr>
</tbody>
</table>

**Exercise 1.4**

Open the simulator and familiarize yourself with the fields shown above. We cannot simulate the Procyon system accurately because the separation distance is much greater than 2.0 AU (the maximum allowed by the simulation). So set the following:

M1 = 1.8, M2 = 1, a = 2.0 AU, e = 0.3 and i = 0 (this assumes the “face-on” observation we based our earlier calculations on – i.e. the orbital plane is perpendicular to our point of view). Press enter and observe the Earth view, the spectral shift and the radial velocity. (should look like the diagram above)

*Can see both stars rotating in near circles. No spectral shift. Radial velocity (towards or away from observer is zero)*
Option E – Astrophysics

Calculate the period of orbit for the system you just programmed. You can assume the masses are given relative to solar mass and the separation distance is equal to half the semi-major axis.

\[ T^2 = \frac{r^3}{M} \]
\[ T = \sqrt{\frac{2.0^3}{2.8}} = 1.69 \text{ years} \]

Now change \( i = 30^\circ \) (this is as close to the actual inclination of 31\(^\circ\) that the simulation allows). What differences do you observe?

Now there is movement towards and away from the observer.

We see a Doppler shift. The graph shows how the radial velocity changes. Where the red and blue lines cross, the radial velocity is zero (motion is perpendicular to observer). Period does not change.

Note: The radial velocity is determined from the Doppler shift in the absorption lines on the spectra. The radial velocity is the component of the star velocity that is parallel to the line of observation. A negative value means the object is moving towards Earth, a positive value indicates movement away from Earth.

Change \( i = 90^\circ \) and note the changes in radial velocity and the stellar spectra. Specifically relate the relative velocity between the two stars to the Doppler shift of the absorption lines.

Here we see a larger shift as the radial velocity has increased.
**Extension**

An inclination angle does NOT affect the characteristics of the binary star system (it is just tilted) but it will affect the assumptions we made about the radius of the Procyon orbit determined in Exercise 3.2.

How would we know that the Procyon orbit might be inclined to our field of view and not perpendicular as originally assumed?

Can you make the correction (considering inclination of 31°) and recalculate the mass of the system. The diagram below should help.

![Diagram](image)

**Q:** Correct your value for the observed semi-major axis for this inclination. What is the new semi-major axis?

\[
\begin{align*}
\text{observed } a \quad &\quad \text{true } a = \frac{\text{observed } a}{\cos (31 \text{ degrees})} \\
14 \text{ AU} \quad &\quad 16.3 \text{ AU} \quad 0.857
\end{align*}
\]

**Q:** Use the corrected semi-major axis to compute the mass of the stars in the Procyon system. How does it compare to your previous mass estimate?

\[
\text{total mass (solar)} = \frac{3}{2} \left( \frac{\text{semi-major axis in AU}}{\text{Period in years}} \right)^3
\]

\[
= \frac{(16.3 \text{ AU})^3}{(41 \text{ yr})^2} \quad \approx \text{about 2.6 solar masses}
\]
Hopefully you have reached a conclusion that the easiest type of spectroscopic binary to observe is one that is exactly "edge-on" to our view. *This type of binary will also behave as an eclipsing binary system.*

![Inclination angle](image)

Eclipsing binary systems have inclination angles close to 90° resulting in one star obscuring light from the other as they pass each other. When the eclipse occurs, the measured light intensity from the system decreases. How much the intensity decreases depends on the relative size and luminosity of the two stars (check your notes). You can see this using the eclipsing binary simulation on the WIKI.

**Exercise 1.5**

Open the simulator and enter the values for Faked-Procyon.

<table>
<thead>
<tr>
<th></th>
<th>Procyon</th>
<th>CV Vel</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Set longitude to 270°</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Inclination</strong></td>
<td>30°</td>
<td>86°</td>
</tr>
<tr>
<td><strong>Mass 1 (M☉)</strong></td>
<td>10</td>
<td>5.6</td>
</tr>
<tr>
<td><strong>Radius 1 (R☉)</strong></td>
<td>4</td>
<td>4.1</td>
</tr>
<tr>
<td><strong>Temperature 1 (K)</strong></td>
<td>7500</td>
<td>18 000</td>
</tr>
<tr>
<td><strong>Mass 2 (M☉)</strong></td>
<td>0.7</td>
<td>5.4</td>
</tr>
<tr>
<td><strong>Radius 2 (R☉)</strong></td>
<td>1</td>
<td>3.9</td>
</tr>
<tr>
<td><strong>Temperature 2 (K)</strong></td>
<td>3270</td>
<td>18 000</td>
</tr>
<tr>
<td><strong>Separation (R☉)</strong></td>
<td>60</td>
<td>34</td>
</tr>
<tr>
<td><strong>Eccentricity</strong></td>
<td>0.4</td>
<td>0</td>
</tr>
</tbody>
</table>

Note values for Procyon have been modified to fit into the ranges of the simulation (the actual separation distance is much greater hence the period appears much shorter than actual in simulation).
Fake-Procyon is NOT an eclipsing binary due to its inclination. It is a visual binary.
Enter the values for CV Vel and observe.

- **System Orientation**
  - longitude: 270.0°
  - inclination: 86.0°

- **Animation and Visualization Controls**
  - pause animation
  - phase: 0.85

- **Star 1 Properties**
  - mass: 5.6 M☉
  - radius: 4.1 R☉
  - temperature: 18000 K

- **Star 2 Properties**
  - mass: 5.4 M☉
  - radius: 2.9 R☉
  - temperature: 18000 K

- **System Properties**
  - separation: 34.00 R☉
  - eccentricity: 0.05

**Presets**
- select a preset -
- reset parameters to match

**Phase**
- Normalized Visual Flux
- System period: 6.9 days
Record the period of the CV Vel system as obtained from the simulation. 6.9 days (indicated on the diagram below)

Key Concept: We detect spectroscopic binary systems by observing Doppler shifts in the spectral lines. This effect is most dramatic if the plane of the orbit is “edge-on” to the observer. Not all spectroscopic binaries are eclipsing binaries.

If one star is orbiting another, it periodically moves toward us and away from us in its orbit, and its spectral lines show blueshifts and redshifts as a result of this motion.

We measure the orbital period of a spectroscopic binary by noting the time it takes the spectral lines to shift back and forth. Except in rare cases, we can calculate the separation of a binary only if we know the actual orbital speeds of the stars. The primary technique for measuring stellar speeds relies on the Doppler effect, but Doppler shifts tell us only the portion of a star’s velocity that is directly toward us or away from us. Orbiting stars generally do not move directly along our line of sight, so their actual velocities can be significantly greater than those we measure through the Doppler effect. The orbits of eclipsing binaries are particularly important to the study of stellar masses because we know the inclination of the system is close to “edge-on” and thus can calculate the actual orbital speeds of the stars.
**Exercise 1.6**
The following diagram shows part of an absorption spectrum for the CV Vel system. There are two sets of absorption lines (one from each star). The peaks correspond to the absorption of magnesium ions (about 448 nm at rest). This value is indicated by the arrow on the diagram. You can see that there has been a shift in each direction (one red and one blue) corresponding to the opposite motions of the two stars in the system.

Determine the shift ($\Delta \lambda$) in the reference peak and calculate the radial velocity of the two stars in the CV Vel system.

Q: What is the shift in wavelength (away from the center) of the line created by each star?

\[
\text{shift} = \text{about 0.2 nm}
\]

Q: What is the orbital speed of each star?

We use the Doppler formula to calculate the radial velocity of each star:

\[
\text{radial velocity} = \frac{\text{shift}}{\text{rest wavelength}} \times (\text{speed of light})
\]

\[
\begin{align*}
0.2 \text{ nm} & \quad \text{shift} \\
448.1 \text{ nm} & \quad \text{rest wavelength} \\
3 \times 10^8 \text{ m/s} & \quad (\text{speed of light}) \\
\end{align*}
\]

\[
133,000 \text{ m/s}
\]
Extension

Assuming the orbit paths in CV Vel are near circular, use the velocity calculated and the period obtained from the simulation to determine the orbital radius.

Q: Assume that both stars in CV Vel move in perfect circles. What is the orbital radius \( R \) for each star? Express your value in AU.

\[
\text{orbital radius } R = \frac{(\text{period}) \times (\text{velocity})}{2 \times \pi}
\]

\[
= \frac{(595,210 \text{ s}) \times (133,000 \text{ m/s})}{2 \times \pi}
\]

\[
= 1.27 \times 10^{10} \text{ m}
\]

\[
= 0.0845 \text{ AU}
\]

The separation distance between the two stars is simply twice the radius you just calculated. This value can be used with Kepler’s 3rd Law to determine the mass of the star system (how does this compare to the values used in the simulation?)

Q: What is the separation between the two stars?
This serves as the semi-major axis \( a \) in Kepler's Third Law when the two objects are comparable in mass to each other.

\[
\text{separation } a = 2 \times R
\]

\[
= 0.169 \text{ AU}
\]

This is about 36 solar radii. The simulation is set to 34.

Q: What is the total mass of the CV Vel system?

\[
\text{total mass (solar)} = \frac{\left(\frac{a^3}{\text{semi-major axis in AU}}\right)}{\left(\frac{\text{Period in years}}{2}\right)^2}
\]

\[
= \frac{(0.169 \text{ AU})^3}{(0.1199 \text{ year})^2} = 13.5 \text{ solar masses}
\]

simulation says 11 solar masses