Review of Galilean-Newtonian Relativity

You know that any measurement of velocity is not absolute, it is relative to the observer’s state of motion.

In any relative velocity problem, it can be helpful to imagine yourself as one of the observers (i.e., you give yourself a velocity of zero relative to that observer).

Situation 1: How do Peter, Stewie and Brian observe each other in the diagram?

Situation 2: How do Peter, Stewie and Brian observe the ball Stewie throws to Brian?
Situation 3: How does Bart observe Peter, Stewie, Brian and the ball?

\[ v = 0 \text{ m/s} \]

\[ 5 \text{ m/s} \]

\[ 15 \text{ m/s} \]

Situation 4: How does Bart observe Peter, Stewie, Brian and the ball?

\[ v = 0 \text{ m/s} \]

\[ 5 \text{ m/s} \]

\[ 15 \text{ m/s} \]
As observers, we consider ourselves to be at the origin of our reference frame (or set of axes).

Consider the diagram (right). Two observers (A and B) are at rest relative to each other and are at the same location in space (they have the same reference frame). At a particular instant in time they both measure the position of point P. Compare the results of each measurement.

Extend: What would happen if both A and B were moving with the same velocities?

Now consider the second diagram (left). We can consider observer A to be at rest and observer B is moving in the x-direction at a constant velocity $u$ (so the relative velocity between A and B is also $u$). How would the measurements of point P differ for each observer?

**Example problem:** Consider a pelican (P) flying in a straight line at a constant velocity of 15 ms$^{-1}$ [E] at an altitude of 25 m. See diagram.

At $t = 0$ s, two observers (A and B) are located at point O with the pelican directly overhead. Observer B is travelling on a bicycle with a constant velocity of 5 ms$^{-1}$ [E]. What position measurements would each observer make of the pelican 5 seconds later?
Practice Problem 1

A train moving with a velocity of 60 km/hr passes through a railroad station at 12.00. Twenty seconds later a bolt of lightning strikes the railroad tracks one km from the station in the same direction that the train is moving. Find the coordinates of the lightning flash as measured by an observer at the station and by the engineer of the train.

Practice Problem 2

A hunter on the ground fires a bullet in the north east direction which strikes a deer 0.25 km from the hunter. The bullet travels with a speed of 1800 km/hr. At the instant when the bullet is fired, an airplane is directly over the hunter at an altitude of one km and is travelling due east with a velocity of 600 km/hr. When the bullet strikes the deer, what are the coordinates as determined by an observer in the airplane?
Derivation of Time – Dilation Formula

A clock is made by sending a pulse of light toward a mirror at a distance L and back to a receiver. Each “tick” is a round-trip to the mirror. The clock is shown at rest in the “Lab” frame in Fig. 1a, or any time it is in its own rest frame. Figure 1b is the way the clock looks to a stationary observer when rocket#1 is moving to the right with velocity \(v\).

Since both legs of the light pulse journey are the same, we’ll just use the one-way times for simplicity so \(t = \frac{1}{2} \text{“tick”}\).

Some notation:
- \(t_0\) = time for light to reach the mirror measured in Rocket#1
- \(t\) = time for light to reach the mirror measured in lab
- \(L\) = distance to mirror (same in both reference frames)

So, the times and distances are related as follows:
- \(L = ct_0\) (from Fig 1a)
- \(L^2 + v^2t^2 = c^2t_0^2\) (From Fig 1b and Pythagorean theorem)

### Eliminate L from the equations:

\[
c^2t_0^2 + v^2t^2 = c^2t^2
\]

\[
c^2t_0^2 = c^2t^2 - v^2t^2
\]

\[
t_0^2 = t^2\left(1 - \frac{v^2}{c^2}\right)
\]

Lab clocks would also appear slow to observers in the rocket. Relativity is symmetrical.

\[
t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}
\]

### Try this. Assume the train is moving at a velocity of 0.5 c.

Dr. Nova (on the train) measures one “tick” of the light clock i.e. the proper time interval, to be exactly 1.0 s. Use the equation to determine what you, a stationary observer would measure the time as.

Answer 1.15 s. What does this imply?

An event observed to be one second long on the train is observed to be 1.15 s long to a stationary observer. This effectively means that your stopwatch runs faster than the stopwatch on the train. Or . . .

**Key concept: Time dilation – moving clocks run slowly**
On the axis below, plot a graph to show how the Lorentz factor varies with $v$ (from zero to $c$).
Time dilation: example problems – do these neatly on a separate page

1. Not very realistic but . . . a simple pendulum oscillates back and forth on a space vehicle. An astronaut on the space vehicle measures the period of the pendulum to be 19.58 seconds (it is a big pendulum). A passing observer in another space ship measures the period to be 26.87 seconds. Determine the relative velocity between the two observers.

2. Consider the new Toronto – Vancouver bullet train that travels at a speed of 0.482 c. A passenger on the bullet train drops a shiny stainless steel ball bearing (r = 1.25 cm) from a height of 1.36 m. What would be the time measured by the passenger, and a stationary observer for the ball bearing to fall to the floor?

3. Rework the above problem using the same given information, but this time the stationary observer drops the ball bearing.

4. Write a summary of the derivation for the time dilation formula – keep it clean and concise – this is a requirement of the course syllabus.

Answers

1. 0.685 c
2. The time for the train passenger would be 0.52 s. The observer would measure 0.59 s.
3. The stationary person would measure 0.52 s, the train passenger would measure 0.59 s (from the point of view of the train passenger, the person dropping the ball is moving). This question leads nicely into the twin paradox we will discuss later.
4. Go back a couple of pages in this handout.

Concept Questions (From Giancoli 6th Edition Q 1-7)

1. You are in a windowless car in an exceptionally smooth train moving at constant velocity. Is there any physical experiment you can do in the train car to determine whether you are moving? Explain.
2. You might have had the experience of being at a red light when, out of the corner of your eye, you see the car beside you creep forward. Instinctively you stomp on the brake pedal, thinking that you are rolling backward. What does this say about absolute and relative motion?
3. A worker stands on top of a moving railroad car, and throws a heavy ball straight up (from his point of view). Ignore air resistance. Will the ball land on the car or behind it?
4. Does the Earth really go around the Sun? Or is it also valid to say that the Sun goes around the Earth? Discuss in view of the first principle of relativity (that there is no best reference frame). Explain.
5. If you were on a spaceship traveling at 0.5c away from a star, at what speed would the starlight pass you?
6. The time dilation effect is sometimes expressed as “moving clocks run slowly.” Actually, this effect has nothing to do with motion affecting the functioning of clocks. What then does it deal with?
7. Does time dilation mean that time actually passes more slowly in moving reference frames or that it only seems to pass more slowly?
**Answers**

1. **No.** Since the windowless car in an exceptionally smooth train moving at a constant velocity is an inertial reference frame and all of the laws of physics are the same in any inertial frame, there is no way for you to tell if you are moving or not.

2. There is no way for you to tell the difference between absolute and relative motion. You always think that you are at rest and that other things around you are moving, but you really can’t tell if maybe you are moving and everything else is at rest.

3. The ball will land on the roof of the railroad car (ignoring air resistance and assuming that the velocity of the train is constant). Both the ball and the car are already moving forward, so when the ball is thrown straight up into the air with respect to the car, it will continue to move forward at the same rate as the car and fall back down to land on the roof.

4. Whether you say the Earth goes around the Sun or the Sun goes around the Earth depends on your reference frame. It is valid to say either one, depending on which frame you choose. The laws of physics, though, won’t be the same in each of these reference frames, since the Earth is accelerating as it goes around the Sun. The Sun is nearly an inertial reference frame, but the Earth is not.

5. If you were in a spaceship traveling at 0.5c away from a star, its starlight would pass you at a speed of c. The speed of light is a constant in any reference frame, according to the 2nd postulate of special relativity.

6. The clocks are not at fault and they are functioning properly. Time itself is actually measured to pass more slowly in moving reference frames when compared to a rest frame.

7. Time actually passes more slowly in the moving reference frame. It is not just that it seems this way, it has actually been measured to pass more slowly, as predicted by special relativity.

**Length contraction: example problems – do these neatly on a separate page**

1. From Kerr and Ruth 3rd edition “Calculate the relative velocity between two inertial observers which produces a 50% reduction in the proper length”.

2. From Kerr and Ruth 3rd edition “A spaceship is travelling away from the Earth with a speed of 0.6 c as measured by an observer on Earth. The rocket sends a light pulse back to Earth every 10 minutes as measured by a clock on the spaceship.
   a. Calculate the distance that the rocket travels between light pulses as measured by the observer on Earth.
   b. If the Earth observer measures the length of the spaceship as 60 m, determine the proper length of the spaceship.”

3. Draw a sketch with appropriate dimensions of how the stationary observer in Question 2 of the previous Time Dilation problem set would measure the dimensions of the falling ball bearing dropped by the passenger on the train.
Answers

1. Use \( L = L_0 \sqrt{1 - \left(\frac{v}{c}\right)^2} \) to get \( v = 0.866 \, c \)

2. a) 10 mins (600 s) on space ship is 750 s on Earth. During this time, the rocket will travel \( 1.35 \times 10^{11} \, m \)
2. b) 75 m

3. vertical dimension would be the same (\( d = 2.50 \, cm \)), since \( v = 0.482 \, c \), the value of \( \gamma \) is about 1.14

the horizontal dimension would then be 2.19 cm

diagram shows actual size

Time Dilation and Length Contraction Problems (From Giancoli 6th Edition Q 1-14)

1. A spaceship passes you at a speed of 0.750c. You measure its length to be 28.2 m. How long would it be when at rest?

   You measure the contracted length. We find the rest length from

   \[
   L = L_0 \left[ 1 - \left(\frac{v}{c}\right)^2 \right]^{\frac{1}{2}};
   \]

   \[
   28.2 \, m = L_0 \left[ 1 - \left(0.750\right)^2 \right]^{\frac{1}{2}}, \text{ which gives } L_0 = 42.6 \, m.
   \]

2. A certain type of elementary particle travels at a speed of \( 2.70 \times 10^8 \, m/s \). At this speed, the average lifetime is measured to be \( 4.76 \times 10^{-6} \, s \). What is the particle’s lifetime at rest?

   We find the lifetime at rest from

   \[
   \Delta t = \frac{\Delta t_0}{\left[ 1 - \left(\frac{v^2}{c^2}\right) \right]^{\frac{1}{2}}};
   \]

   \[
   4.76 \times 10^{-6} \, s = \frac{\Delta t_0}{\left[ 1 - \left(\frac{(2.70 \times 10^8 \, m/s)^2}{c^2} \right) \right]^{\frac{1}{2}}}, \text{ which gives } \Delta t_0 = 2.07 \times 10^{-6} \, s.
   \]
3. Lengths and time intervals depend on the factor $\sqrt{1-v^2/c^2}$ according to the theory of relativity. Evaluate this factor for speeds of: (a) $v = 20,000 \text{ m/s}$ (typical speed of a satellite); (b) $v = 0.020c$; (c) $v = 0.200c$; (d) $v = 0.95c$; (e) $v = 0.98c$; (f) $v = 0.999c$.

(a) \[ \left[ 1 - \left( \frac{v}{c} \right)^2 \right]^{\frac{1}{2}} = \left[ 1 - \left( \frac{20,000 \text{ m/s}}{3.00 \times 10^8 \text{ m/s}} \right)^2 \right]^{\frac{1}{2}} = 1.00. \]

(b) \[ \left[ 1 - \left( \frac{v}{c} \right)^2 \right]^{\frac{1}{2}} = \left[ 1 - (0.020)^2 \right]^{\frac{1}{2}} = 0.9998. \]

(c) \[ \left[ 1 - \left( \frac{v}{c} \right)^2 \right]^{\frac{1}{2}} = \left[ 1 - (0.200)^2 \right]^{\frac{1}{2}} = 0.980. \]

(d) \[ \left[ 1 - \left( \frac{v}{c} \right)^2 \right]^{\frac{1}{2}} = \left[ 1 - (0.95)^2 \right]^{\frac{1}{2}} = 0.312. \]

(e) \[ \left[ 1 - \left( \frac{v}{c} \right)^2 \right]^{\frac{1}{2}} = \left[ 1 - (0.98)^2 \right]^{\frac{1}{2}} = 0.199. \]

(f) \[ \left[ 1 - \left( \frac{v}{c} \right)^2 \right]^{\frac{1}{2}} = \left[ 1 - (0.999)^2 \right]^{\frac{1}{2}} = 0.0447. \]

4. If you were to travel to a star 125 light-years from Earth at a speed of $2.50 \times 10^8 \text{ m/s}$, what would you measure this distance to be?

You measure the contracted length:

\[
L = L_0 \left[ 1 - \left( \frac{v}{c} \right)^2 \right]^{\frac{1}{2}} = (125 \text{ ly}) \left[ 1 - \left( \frac{(2.50 \times 10^8 \text{ m/s})}{(3.00 \times 10^8 \text{ m/s})} \right)^2 \right]^{\frac{1}{2}} = 69.1 \text{ ly}. 
\]
5. What is the speed of a pion if its average lifetime is measured to be $4.10 \times 10^{-8} \text{ s}$? At rest, its average lifetime is $2.60 \times 10^{-8} \text{ s}$. We determine the speed from the time dilation:

$$
\Delta t = \Delta t_0 \left[1 - \left(\frac{v^2}{c^2}\right)^{\frac{1}{2}}\right];
$$

$$
4.10 \times 10^{-8} \text{ s} = \frac{(2.60 \times 10^{-8} \text{ s})}{\left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}}, \text{ which gives } v = 0.773c.
$$

6. In an Earth reference frame, a star is 82 light-years away. How fast would you have to travel so that to you the distance would be only 35 light-years?

We determine the speed from the length contraction:

$$
L = L_0 \left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}};
$$

$$
35 \text{ ly} = (82 \text{ ly}) \left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}, \text{ which gives } v = 0.90c.
$$

7. At what speed $v$ will the length of a 1.00-m stick look 10.0% shorter (90.0 cm)?

We determine the speed from the length contraction:

$$
L = L_0 \left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}};
$$

$$
0.900 \text{ m} = (1.00 \text{ m}) \left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}, \text{ which gives } v = 0.436c.
$$
8. Escape velocity from the Earth is $40,000 \text{ km/h}$. What would be the percent decrease in length of a 95.2-m-long spacecraft traveling at that speed?

We convert the speed:

$$\frac{(40,000 \text{ km/h})}{(3.6 \text{ ks/h})} = 1.11 \times 10^4 \text{ m/s}.$$ 

The length contraction is given by

$$L = L_0 \sqrt{1 - \left(\frac{v}{c}\right)^2};$$

$$1 - \frac{L}{L_0} = 1 - \sqrt{1 - \left(\frac{v}{c}\right)^2} = 1 - \sqrt{1 - \left[\frac{1.11 \times 10^4 \text{ m/s}}{3.00 \times 10^8 \text{ m/s}}\right]^2}$$

$$= 7 \times 10^{-10} \text{ or } 7 \times 10^{-8} \%.$$

9. Suppose a news report stated that starship Enterprise had just returned from a 5-year voyage while traveling at 0.84$c$. (a) If the report meant 5.0 years of Earth time, how much time elapsed on the ship? (b) If the report meant 5.0 years of ship time, how much time passed on Earth?

In the Earth frame, the clock on the Enterprise will run slower.

(a) We find the elapsed time on the ship from

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}};$$

$$5.0 \text{ yr} = \frac{\Delta t_0}{\sqrt{1 - (0.84)^2}}$$

which gives $\Delta t_0 = 2.7 \text{ yr}.$

(b) We find the elapsed time on the Earth from

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}$$

$$= \frac{(5.0 \text{ yr})}{\sqrt{1 - (0.84)^2}} = 9.2 \text{ yr}.$$
10. A certain star is 10.6 light-years away. How long would it take a spacecraft traveling 0.960c to reach that star from Earth, as measured by observers: (a) on Earth, (b) on the spacecraft? (c) What is the distance traveled according to observers on the spacecraft? (d) What will the spacecraft occupants compute their speed to be from the results of (b) and (c)?

(a) To an observer on Earth, 10.6 ly is the rest length, so the time will be

\[ t'_{\text{Earth}} = \frac{L_0}{v} = \frac{10.6 \text{ ly}}{0.960c} = 11.0 \text{ yr.} \]

(b) We find the dilated time on the spacecraft from

\[ \Delta t = \frac{\Delta t_0}{\left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}}; \]

\[ 11.0 \text{ yr} = \frac{\Delta t_0}{\left[1 - (0.960c)^2\right]^{\frac{1}{2}}}, \text{ which gives } \Delta t_0 = 3.09 \text{ yr.} \]

(c) To the spacecraft observer, the distance to the star is contracted:

\[ L = L_0 \left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}} = (10.6 \text{ ly})\left[1 - (0.960c)^2\right]^{\frac{1}{2}} = 2.97 \text{ ly.} \]

(d) To the spacecraft observer, the speed of the spacecraft is

\[ v = \frac{L}{\Delta t} = \frac{(2.97 \text{ ly})}{3.09 \text{ yr}} = 0.960c, \text{ as expected.} \]

11. A friend speeds by you in her “Ferrari” spacecraft at a speed of 0.660c. It is measured in your frame to be 4.80 m long and 1.25 m high. (a) What will be its length and height at rest? (b) How many seconds would you say elapsed on your friend’s watch when 20.0 s passed on yours? (c) How fast did you appear to be traveling according to your friend? (d) How many seconds would she say elapsed on your watch when she saw 20.0 s pass on hers?

(a) You measure the contracted length. We find the rest length from

\[ L = L_0 \left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}; \]

\[ 4.80 \text{ m} = L_0 \left[1 - (0.660c)^2\right]^{\frac{1}{2}}, \text{ which gives } L_0 = 6.39 \text{ m.} \]

Distances perpendicular to the motion do not change, so the rest height is 1.25 m.

(b) We find the dilated time in the spacecraft from

\[ \Delta t = \frac{\Delta t_0}{\left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}}; \]

\[ 20.0 \text{ s} = \frac{\Delta t_0}{\left[1 - (0.660c)^2\right]^{\frac{1}{2}}}, \text{ which gives } \Delta t_0 = 15.0 \text{ s.} \]

(c) To your friend, you moved at the same relative speed: 0.660c.

(d) She would measure the same time dilation: 15.0 s.
12. How fast must an average pion be moving to travel 15 m before it decays? The average lifetime, at rest, is $2.6 \times 10^{-8}$ s.

In the Earth frame, the average lifetime of the pion will be dilated:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}.$$  

The speed as a fraction of the speed of light is

$$\frac{v}{c} = \frac{d}{c} \frac{1}{\Delta t} \frac{1}{\Delta t_0}.$$  

$$\frac{v}{c} = \frac{(15 \text{ m})}{(3.00 \times 10^8 \text{ m/s})} \frac{1}{(2.60 \times 10^{-8} \text{ s})},$$

which gives $v = 0.887c = 2.66 \times 10^8 \text{ m/s}$. 